Self-assessment *continued*

7. Derive the expression,

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

relating average molecular kinetic energy and absolute temperature of a gas.

8. State **Avogadro's law** and prove your statement.

9. (a) Consider an ideal gas enclosed in a cylinder by a piston of area (A). What is the **work done** when the piston moves over a small distance Δx as a result of the gas pressure (p)? Assume that the change in volume is so small that the pressure remains constant. What is the work done in terms of the volume change of the gas?
   (b) What is the expression for the total work done by the gas for a **finite** expansion in which the volume increases from V_1 to V_2? What is the expression if the expansion occurs at **constant pressure?**

10. (a) State the **first law of thermodynamics**.
    (b) Explain what is meant by **internal energy** in the context of the first law.
    (c) How can the internal energy of a system be (i) **increased**, (ii) **decreased?**
    (d) What is the alternative form of the first law which is based on the change of internal energy of a system?

11. (a) What is an **isolated** system?
    (b) Why is the internal energy of such a system constant?
    (c) What is an **adiabatic** process?
    (d) What do you conclude by applying the first law of thermodynamics to a system which undergoes an adiabatic process?

12. (a) Why can a gas have an infinite number of heat capacities?
    (b) Define (i) the **molar heat capacity at constant volume** C_v
    and (ii) the **molar heat capacity at constant pressure** C_p, of a gas.
    (c) Explain why C_p is greater than C_v.
    (d) Give an expression for the amount of heat ΔQ which must be supplied to n moles of an ideal gas so as to produce a temperature rise ΔT if the heating is performed (i) at **constant volume**, and (ii) at **constant pressure**.

13. The state of a gas can be changed by each of the following processes: **isothermal; isovolumetric; isobaric; adiabatic**. For each of these:
    (a) State the condition under which the change occurs.
    (b) Show the change on a pressure–volume graph.

SECTION B
Quantitative assessment

(***Answers:*** 1.5×10^{-4}; 2.7×10^{-4}; 2.1×10^{-3}; 8.6×10^{-3}; 0.010; 0.026; 0.80; 1.3; 60; 120; 210; 240; 240; 480; 480; 480; 480; 480; 480; 520; 520; 730; 830; 920; 1.6×10^3; 2.0×10^3; 2.5×10^3; 2.6×10^3; 7.0×10^3; 1.0×10^4; 2.1×10^4; 2.3×10^4; 2.3×10^5; 4.8×10^{23}.)

1. (a) A fixed mass of gas has a volume of 3000 cm^3 at a pressure of 1.0×10^5 Pa. Calculate its volume if the pressure increases to 2.0×10^6 Pa with the temperature remaining constant.
   (b) A fixed mass of gas has a volume V when the temperature is 127 °C. To what temperature must the gas be raised so that its volume increases to 2.75 V with the pressure remaining constant?
   (c) A fixed mass of gas has a volume of 0.02 m^3 at a temperature of 44 °C and a pressure of 2.02×10^5 Pa. Find the new volume of the gas at standard temperature and pressure (i.e. 0 °C and 1.01×10^5 Pa).

2. (a) At a particular instant five oxygen molecules have speeds of 450, 475, 480, 495 and 510 m s^{-1}.
   Calculate their:
   (i) mean speed
   (ii) mean-square speed and
   (iii) root-mean-square speed.
   (b) Calculate the r.m.s. speed for the molecules of a gas of density 1.3 kg m^{-3} at a pressure of 1.0×10^5 Pa.

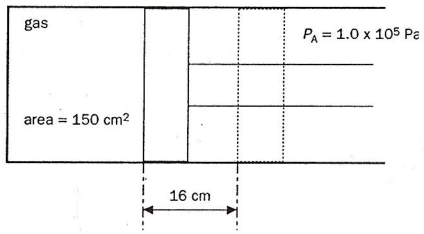
4. A sealed can of volume 2×10^4 cm^3 contains gas at a pressure of 1×10^5 Pa and a temperature of 27 °C. Assuming the gas to be ideal, calculate:
   (a) the number of moles of gas in the can (given that the universal molar gas constant, R = 8.31 J mol^{-1} K^{-1})

Self-assessment *continued*

   (b) the number of gas molecules in the can (given that the Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1})
   (c) the mass of gas in the can if its relative molecular mass is 32
   (d) the gas density
   (e) the r.m.s. speed of the gas molecules.

4. The total translational kinetic energy of a certain mass of an ideal gas at a temperature of 57 °C is 550 J. If the relative molecular mass of the gas is 2.0, calculate:
   (a) the total kinetic energy of the gas molecules at a temperature of 280 °C
   (b) the mass of gas present and
   (c) the r.m.s. speed of the molecules at 280 °C
   (given that the universal molar gas constant, R = 8.3 J mol^{-1} K^{-1}).

5.

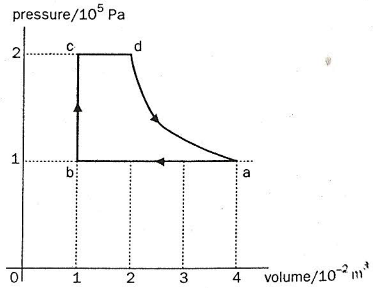


The diagram shows a sample of gas enclosed in a cylinder by a frictionless piston of area 150 cm^2. When 300 J of energy is supplied to the gas, it expands against a constant atmospheric pressure of 1.0×10^5 Pa and pushes the piston out a distance of 16 cm along the cylinder. Calculate:
   (a) the work done by the gas and
   (b) the increase in internal energy of the gas.

6. When 10 cm^3 of water is boiled at 100 °C and at an atmospheric pressure of 1.0×10^5 Pa, 1.6×10^4 cm^3 of steam is produced. Calculate:
   (a) the mass of water boiled
   (b) the heat energy needed to produce the vaporisation
   (c) the external work done during the vaporisation
   (d) the increase in internal energy.
   (Density of water = 1.0×10^3 kg m^{-3}; specific latent heat of vaporisation of steam = 2.26×10^6 J kg^{-1}.)

7. An insulated, freely extensible vessel contains 0.5 mol of oxygen at 273 K. If atmospheric pressure is taken as 1.0×10^5 Pa and the principal molar heat capacity at constant pressure of oxygen is 29 J mol^{-1} K^{-1} calculate:
   (a) the heat energy needed to raise the temperature of the gas to 323 K
   (b) the volume increase of the gas produced by the temperature rise
   (c) the external work done by the gas
   (d) the increase in the internal energy of the gas
   (e) the heat energy needed to cause the same temperature rise at constant volume.
   (Universal molar gas constant, R = 8.3 J mol^{-1} K^{-1}.)

8.



The indicator (pV) diagram shows an energy cycle for **one mole** of an ideal gas which is: cooled at constant pressure (a to b), heated at constant volume (b to c), heated at constant pressure (c to d) and finally returned to its original state by an isothermal expansion (d to a).

Calculate:
   (a) the gas temperature at a, b, c and d
   (b) the heat energy removed or supplied during the processes a to b, b to c, and c to d
   (c) the net work done in 1 cycle (an estimate).
   (Universal molar gas constant, R = 8.3 J mol^{-1} K^{-1}; molar heat capacity at constant pressure of the gas, C_p = 29 J mol^{-1} K^{-1}.)

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Quantitative assessment

(1)(a) $V_I = 3000\,cm^3 = 3\times10^{-3}\,m^3$

$P_I = 1\times10^5\,Pa$

$P_F = 2\times10^6\,Pa$

$T = constant$

$P_I V_I = P_F V_F$

$\frac{1\times10^5 \times 3\times10^{-3}}{2\times10^6} = \underline{\underline{1.5\times10^{-4}\,m^3}}$

(b) Fixed Mass.

$V = V$ when $T = 127$

$V = 2.75V$ $T =$

$P = constant$

$\frac{V_I}{T_I} = \frac{V_F}{T_F} \rightarrow \frac{1}{127} = \frac{2.75}{T}$

BUT T IN K $\frac{1}{400} = \frac{2.75}{T}$

$T = 1100\,K$

$= \underline{\underline{827\,°C}}$

(c) $V_I = 0.02\,m^3$

$M = constant$

$T_I = 44°C = 317K$

$P_I = 2.02\times10^5\,Pa$ $V_F = ??$

$\frac{P_1V_1}{T_1} = \frac{P_F V_F}{T_F}$ $\frac{2.02\times10^5 \times 0.02}{317} = \frac{1.01\times10^5 \times V_F}{273}$

$V_F = 0.034\,m^3$

(B)

(2)

(a) (i) Mean $= \underline{482\ ms^{-1}}$

(ii) mean-square speed $= 232730\ m^2s^{-2}$ $\underline{\underline{2.3\times10^5 m^2s^{-2}}}$

(iii) root mean square speed $= 482.4\ ms^{-1}$

$= \underline{\underline{482 ms^{-1}}}$

(b) r.m.s

$\rho = 1.3\ kg\,m^{-3}$

$p = 1.0\times10^5\ Pa$

$$P = \frac{1}{3}\rho\bar{c}^2$$

$$1.01\times10^5 = \frac{1}{3}\times1.3\times\bar{c}^2$$

$$\bar{c}^2 = 233076.9$$

$$= 2.33\times10^5\ m^2s^{-2}$$

$$\sqrt{\bar{c}^2} = 482.78$$

$$= \underline{\underline{483 ms^{-1}}}$$

labelled as 4 (3) $V = 2\times10^4 cm^3 = 0.02 m^3$

$P = 1\times10^5\ Pa$

$T = 27°C = 300K$

(a) $R = 8.31\ J mol^{-1} K^{-1}$

$PV = nRT$

$$\frac{PV}{RT} = n \qquad \frac{1\times10^5\times0.02}{8.31\times300} = \underline{\underline{0.802\ moles}}$$

(b) number of gas molecules $\qquad 0.802\times N_A = \underline{\underline{4.83\times10^{23}\ molecules}}$

$N_A = 6.02\times10^{23}$

Ⓒ

③ ctd (c) RRM = 32

MASS = ?

1 mole = 32g

0.802 moles = 25.664 g

= 26 g

= $\underline{\underline{0.026\ kg}}$

(d) $\rho = \frac{m}{V} = \frac{0.026}{0.02} = \underline{\underline{1.283\ kgm^{-3}}}$

$= \underline{\underline{1.3\ kgm^{-3}}}$

(e) RMS

$P = \frac{1}{3}\rho\bar{c}^2$

$1\times10^5 = \frac{1}{3}\times1.3\times\bar{c}^2$

$\sqrt{\bar{c}^2} = \sqrt{230769}$

$= 480.32$

$= \underline{\underline{480\ ms^{-1}}}$

④ T = 57°C = 330K

KE = 550J

RRM = 2

KE ∝ T

(a) KE @ T = 280°C = 553 K

$\frac{553}{330} = 1.675$

550 × 1.675 = 921.6

= $\underline{\underline{922J}}$